MR2722418 (2011m:01002) 01A20
Lelgemann, Dieter (D-TUB)

★**Die Erfindung der Messkunst. (German) [The discovery of mensuration]**
Angewandte Mathematik im antiken Griechenland. [Applied mathematics in ancient Greece]
With a foreword by Eberhard Knobloch.
Wissenschaftliche Buchgesellschaft, Darmstadt, 2010. 285 *pp.* €39.90. *ISBN* 978-3-534-23118-8

The ambition of this book is to correct the misunderstandings of historians of ancient exact science; the tool to be applied is the ability of the engineer to make use of numerical data. More specifically, the author intends to show that heliocentricity was widely adhered to in antiquity, and that ancient astronomical and geographical measurements were astonishingly precise. The key characters are Ptolemy (*Almagest* as well as *Geography*) and a very shadowy Eratosthenes—shadowy because all evidence is indirect, most of it taken from Strabo, and because the author attributes to Eratosthenes many data from Strabo for which Strabo gives no source. The intended audience consists of historians as well as the general public.

Unfortunately, the author's use of the sources—numerical data as well as texts—falls severely short of what would be needed for the construction of a convincing argument. This starts at the very first page of his text (p. 17), where the author wonders why nobody has seen that Ptolemy's table of chords (*Almagest* I.11) is actually a table of sines. He gives support to this claim by quoting the first 4 entrances $(\frac{1}{2}^{\circ}, 1^{\circ}, 1\frac{1}{2}^{\circ}, 2^{\circ})$ and showing how well the purported chords coincide with the corresponding sines. Unfortunately he does not show that they coincide much better with the doubled sines of the half-angles, that is, as Ptolemy as well as the translation used by the author (Manitius) states, with the chord $(\sin 2^{\circ} = 0.034899, 2 \cdot \sin 1^{\circ} = 0.034905$; Ptolemy's sexagesimal value corresponds to 0.034907). If the author had gone on to 180°, the error would of course have been glaring.

Equally wrong is another marvelous mathematical discovery—pp. 232–234, stressed again in the conclusion, p. 260: a "construction of the ellipse with ruler and compass, that is, a purely geometric explanation of the equant model" (it is actually, of course, only a pointwise construction of the ellipse, thus nothing sensational). It is stated that a point on an epicycle moving with the same angular velocity as the motion of the centre of the epicycle on the deferent but in the opposite direction describes an ellipse. The claim is not proved, but the proof is simple: If the radii of the deferent and the epicycle are, respectively, R and r, and the angular position of the centre of the epicycle from the centre of the deferent is φ, then (with an adequate choice of x-axis direction) the coordinates of the moving point are $(x, y) = ((R + r) \cdot \cos \varphi, (R - r) \cdot \sin \varphi)$, which evidently describes an ellipse with semi-axes $R + r$ and $R - r$ (if $R = r$, we get that linear motion produced by two circular motions which al-Tūsī and Copernicus used to get rid of the equant). However, if φ varies uniformly, the speed of the point is symmetric about both axes, for which reason the whole construction can never have anything to do with the equant unless this irregular motion is smuggled in beforehand, which the author says nothing about. Instead he claims that if $r = 0$, then we get the equant model. The equant is never explained, only a quotation of Ptolemy's words from the *Almagest* on p. 232 (diagrams, here as regularly, are not explained, only shown) is offered.

A perplexing absence of mathematical understanding also shows up when the author discusses the doubling of the cube: on p. 97 he wonders how Hippocrates was able to find the "strange" condition $a : b = 1 : 2$; on p. 99, he also finds this numerical expression of doubling (now written $b = 2a$) "strange" (*seltsam*).

The use of textual evidence is no less problematic. A large number of quotations are used in the argument: Ptolemy according to Manitius' translation, others often (nothing more specific is said) translated anew for the author by Eberhard Knobloch and Andreas Kleineberg—actually well translated where the reviewer checked. The problem concerns the way the quotations are used by the author. For instance, Strabo's *Geography* (2.1.6; the author writes I.6) contains this passage (translated by H. L. Jones, slightly corrected in order to agree with the German), quoted on pp. 198–199:

> Neither does this statement of Patrocles lack plausibility, namely, that those who made the expedition with Alexander acquired only cursory information about everything, but Alexander himself had accurate investigations made, since the men best acquainted with the country had described the whole of it for him.

The author adds to "described" the explanation "[surveyed]" (*aufgemessen*), even though the data in question concern many regions never reached by Alexander's army, and adds that "Alexander and his friends were indeed taught by Aristotle, certainly also in military surveying"; it is thus supposed that Alexander's Macedonian companions, who had never been there before, were those "best acquainted" with India. The fancy about Aristotle is already found in a rhetorical question on p. 51: "Would Aristotle's teaching of the military surveying engineers have neglected to explain to the bematists (step counters)/navigators the use of the astronomical methods of his friend Calippus?"

Plato's request from the *Republic* (VII, 528a-b; the author does not specify the locus) that stereometry be studied before astronomy (namely because bodies have to be studied *in their essence as solids* before one engages with bodies that rotate) is read on p. 225 as a claim that one has to take the distances of the heavenly bodies into account, in contradistinction to what is done in the Eudoxos-Aristotle model of homocentric spheres (similarly but briefer on p. 43).

In order to show that "Ancient astronomical and geographical measurements were astonishingly precise", various tricks are used. When Eratosthenes' values for east-west-distances are much too large, it is supposed that they refer to the distance between the corresponding meridians at the equator; when they fit, they are taken at face value. Most astonishing is the way Eratosthenes is shown to have measured the circumference of the earth as exactly 40,000 km. This comes from identifying Eratosthenes' stadium with 600 Gudea feet. This foot is claimed to be 0.26455 m; no reference is given, but the only item in the bibliography which proposes itself as the source [R. C. A. Rottländer, "Vormetrische Längeneinheiten", vormetrische-laengeneinheiten.de] states it to be $265.10 \pm 0,576$ mm. the author appears to have produced his value backwards from the result he wants. It may be added that the status of the Gudea foot itself is highly dubious (it adds to the distance between the two delimiting strokes on the ruler of the Gudea statue at the Louvre one of the two bits outside the strokes, slightly shorter than the 15 "finger breadths" marked inside); equally dubious is probably the idea of a common metrology for north-west European megalithic monuments, Mesopotamia, Egypt, and the Greco-Roman world, which the author borrows from Rottländer.

Heron would seem an unavoidable figure in a book on ancient mensuration. He *is* discussed, but the author appears only to know him from Moritz Cantor's discussion in the 1894 edition of his *Vorlesungen* (even though the critical edition from 1899–1914 is provided with German translations; p. 74 mentions "an appendix to Heron's writing (Heiberg 1972)", perhaps the reprint, but the item is not in the bibliography and the reference is thus likely to be indirect). No wonder the author does not distinguish Heron's genuine works from compilations that were wrongly attributed to him, clearly separated from his genuine works by Heiberg.

Even the aim to write for a general public is not attained. Too often, the variables of equations or the quantities indicated in diagrams are left unexplained; a reader who does not know modern geodesic conventions will not be able to follow the argument. Similarly, one who does not already know Ptolemy—e.g., that what in Heiberg's critical edition, by Manitius, and in later discussions of the work is expressed 1^{P} is $\frac{1}{120}$ of the diameter of the basic circle, taken itself to be 120—will have a hard time (on p. 17 1^{P} is simply stated to be $\frac{1}{60}$; when the expression turns up later on pp. 243–255, this makes absolutely no sense).

With these examples in mind, the reader should check everything relevant to a result presented, from mathematical calculations and metrology to the relation between quotations, their non-quoted textual contexts, and what is derived from them by the author; even general historical information should be verified. Since references are often absent (even from source citations except from Strabo and Ptolemy), such validation will often be arduous.

Reviewed by *Jens Høyrup*